

# Comment on “Matter-Wave Interferometry of a Levitated Thermal Nano-Oscillator Induced and Probed by a Spin”

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References [1, 2] propose an experiment involving the center of mass (COM) position of a nanodiamond and the spin of its NV center to demonstrate “the interference between spatially separated states of the center of mass of a mesoscopic harmonic oscillator ... by coupling it to a spin and performing solely spin manipulations”. The nanodiamond is held in a harmonic potential and also feels the force from gravity. The idea is to use a spatially varying magnetic field  $\vec{B} = B_0(-x\hat{x} - y\hat{y} + 2z\hat{z})$  to couple the spin and COM degrees of freedom. In this comment, we show that the results of the proposed experiment do not result from an entanglement between the spin and the COM position, and, hence, a measurement on the spin part of the wave function can not give information about the COM separation of the  $\pm 1$  states.

The conceptual problem is that the nanodiamond is not oscillating about the  $z = 0$  point of the harmonic potential but about the shifted position,  $-z_0$ , due to gravity. The spatial shift is of order  $10^{-9}$  m. To compare, the separation of the  $|\pm 1\rangle$  states due to the difference in the force on them from the spatially dependent magnetic field is of order  $10^{-13}$  m. At the shifted position,  $\vec{B}$  is *nonzero* which leads to a trivial phase accumulation between the  $+1$  and  $-1$  states. It is exactly this phase difference which leads to Eq. (11,10) in Refs. [1, 2]. This phase difference can be simply cancelled using a small magnetic field offset and is not the result of entanglement between the position and spin operators; we note that the constant term in the magnetic field, Eq. (1) of Refs. [1, 2], is specifically not included in their discussion although it leads to exactly the same kind of phase difference.

The Hamiltonian, Eq. (4) of Refs. [1, 2], is rewritten using the *shifted* COM coordinate  $\tilde{z} \equiv z + z_0$  as

$$H = DS_z^2 + \hbar\omega_z\tilde{c}^\dagger\tilde{c} - 2\lambda S_z(\tilde{c}^\dagger + \tilde{c}) + \sqrt{\frac{2m\omega_z}{\hbar}}z_0 2\lambda S_z - E_s \quad (1)$$

where  $\tilde{z} = \sqrt{\hbar/(2m\omega_z)}(\tilde{c}^\dagger + \tilde{c})$ ,  $S_z$  is in units of  $\hbar$ ,  $DS_z^2$  is from the NV center anisotropic spin interaction,  $\tilde{c}, \tilde{c}^\dagger$  are the lowering and raising operators in the shifted coordinate system  $\tilde{z}$ ,  $\omega_z$  is the COM oscillation frequency,  $\lambda = B_0 g_{NV} \mu_B \sqrt{\hbar/(2m\omega_z)}$ ,  $B_0$  is from the spatially varying magnetic field above,  $g_{NV}$  is the Landé factor of the NV center,  $\mu_B$  is the Bohr magneton,  $z_0 \equiv g \cos(\theta)/\omega_z^2$ ,

$\theta$  is the angle between the  $z$ -axis and vertical, and the constant  $E_s = (1/2)m\omega_z^2 z_0^2$ . For the discussion below, the first three terms will be grouped into  $H_1$ , the fourth term will be defined as  $H_2$ , and the fifth term is a constant and, thus, can be dropped.

The time propagation of the wave function can be found exactly. Most importantly, the operators  $H_1$  and  $H_2$  commute. This means the wave function propagation can be written exactly as

$$\Psi(t) = \exp(-iH_2 t/\hbar) \exp(-iH_1 t/\hbar) \Psi(0) \quad (2)$$

where  $\Psi(0) = \psi_0(\tilde{z})(|+1\rangle + |-1\rangle)/\sqrt{2}$  is an initial spatial function times the symmetric combination of spins  $+1$  and  $-1$ . One can use the methods in Refs. [1, 2] to solve for the time dependent wave function or one can decompose  $\Psi(0)$  into the eigenstates of the  $H_1$  operator. After an integer  $N$  periods,  $t = 2\pi N/\omega_z$ , the

$$e^{-iH_1 t/\hbar} \Psi(0) = e^{iN\eta} \Psi(0) \quad (3)$$

where  $\eta = 8\pi\lambda^2/(\hbar\omega_z)^2 - 2\pi D/(\hbar\omega_z)$ . Thus, the part of the Hamiltonian that contains both the  $S_z$  and the  $\tilde{z}$  operators, which is the only part of  $H$  that can entangle the spin and COM degrees of freedom, gives *no effect* on the wave function after an integer number of periods.

However, the term from  $H_2 = \sqrt{2m\omega_z/\hbar} z_0 2\lambda S_z$  gives

$$e^{-iH_2 t/\hbar} \Psi(0) = e^{-iN\phi/2} \psi_0(z) \frac{|+1\rangle + e^{iN\phi} |-1\rangle}{\sqrt{2}} \quad (4)$$

after  $N$  periods, where  $\phi = 8\pi\lambda z_0 \sqrt{2m\omega_z/\hbar}/(\hbar\omega_z)$ . Evaluating  $\phi$  and  $\Delta\phi_{\text{grav}}$  in Eq. (10) of Ref. [1] or Eq. (9) of Ref. [2], one can show that  $\phi = \Delta\phi_{\text{grav}}$ . Thus, the main result of Ref. [1], Eq. (9), or the equivalent Eq. (8) of Ref. [2], is exactly obtained in Eq. (4). Since  $H_2$  is proportional to  $S_z$ , has no dependence on  $\tilde{z}$ , and commutes with  $H_1$ , it *can not* contain information about the spatial degrees of freedom. Also, the  $H_2$  can be exactly cancelled by a uniform magnetic field which means the measurement is not probing the separation of the  $\pm 1$  states. Thus, the proposed measurement would not give information about “spatially separated states of the center of mass of a mesoscopic harmonic oscillator”.

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